

DIRECTORATE OF DISTANCE EDUCATION
MAHARSHI DAYANAND UNIVERSITY, ROHTAK



New Scheme of Examination

Master of Science (Mathematics)
Two Year Programme (Annual)

First Year (Previous)

Paper	Nomenclature	Marks
MM1001	Advanced Abstract Algebra	100
MM1002	Real Analysis	100
MM1003	Topology	100
MM1004	Programming in C	100
MM1005	Differential Equations	100

Second Year (Final)

Paper	Nomenclature	Marks
MM2001	Integration theory and Functional Analysis	100
MM2002	Partial Differential Equations and Mechanics	100
MM2003	Complex Analysis	100
	Choose either of the group:-	
	Group-I (Pure Group)	
MM2004	Advanced Discrete Mathematics	100
MM2005	Analytical Number Theory	100
	OR	
	Group-II (Applied Group)	
MM2006	Mechanics of solids	100
MM2007	Fluid Dynamics	100

MASTER OF SCIENCE (MATHEMATICS)

M.Sc. (Previous) ADVANCED ABSTRACT ALGEBRA PAPER CODE: MM1001

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Section I

Groups: Normal and subnormal series Composition series, Jordan–Holder theorem. Solvable groups. Nilpotent groups. Conjugate elements, class equation for a finite group. Sylow p-subgroup, Sylow’s theorems and their simple applications. Survey of finite groups upto order 15.

Section II

Canonical Forms: Similarity of linear transformations. Invariant subspaces Reduction to triangular form. Nilpotent transformations. Index of nilpotency. Invariants of nilpotent transformations. The primary decomposition theorem. Jordan blocks and Jordan forms.

Section III

Field Theory. Algebraic and transcendental extensions. Separable and inseparable extensions. Normal extensions. Perfect fields. Finite fields. Primitive elements. Algebraically closed fields. Automorphism of extensions. Galois extensions. Fundamental Theorem of Galois Theory. Solution of polynomial equations by radicals. Insolvability of the general equation of degree 5.

Section IV

Simple modules. Schaur’s Lemma. Free modules fundamental structure, theorem of finitely generated modules over principal, ideal domain and its applications to finitely generated abelian groups. Noetherian and Artinian modules and rings. Hilbert basis theorem. Wedderburn Artin theorem. Uniform modules. Primary modules and Noether-Lasker theorem.

REAL ANALYSIS PAPER CODE: MM1002

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit I

Sequence and series of functions, pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass’s M test, Abel’s and Dirichlet’s tests for uniform convergence, uniform convergence and continuity, Uniform convergence and Riemann-Stieltjes Integration, uniform convergence and differentiation,. Weierstrass Approximation theorem, Power series, Uniqueness theorem for power series, Abel’s and Tauber’s theorems.

Unit II

Functions of several variables, linear transformations, derivatives in an open subset of \mathbb{R}_n , Partial derivatives, Higher order differentials, Taylor’s theorem. Explicit and Implicit functions. Implicit function theorem and inverse function theorem. Change of variables. Extreme values of explicit and stationary values of implicit functions. Lagrange’s multipliers method. Jacobian and its properties.

Unit III

Definition and existence of Riemann-Stieltjes integral, Properties of the integral, Integration and differentiation, The fundamental theorem of calculus, Integration of vector-valued functions, Rectifiable curves. Set functions, intuitive idea of measure, Elementary properties of measure, Measurable sets and their fundamental properties. Lebesgue measure of sets of real numbers, Algebra of measurable sets; Borel sets, Equivalent formulation of measurable sets in terms of open, Closed, F_σ and G_δ sets, Non measurable sets.

Unit IV

Measurable functions and their equivalent formulations. Properties of measurable functions. Approximation of measurable functions by sequences of simple functions, Measurable functions as nearly continuous functions, Egoroff's theorem, Lusin's theorem, Convergence in measure and F. Hiesz theorem for convergence in measure. Almost uniform convergence. Shortcomings of Riemann Integral, Lebesgue Integral of a bounded function over a set of finite measure and its properties. Lebesgue integral as a generalisation of Riemann integral, Bounded, convergence theorem, Lebesgue theorem regarding points of discontinuities of Riemann integral functions, Integral of non-negative functions, Fatou's Lemma, Monotone convergence- Theorem, General Lebesgue Integral, Lebesgue convergence theorem.

Unit V

Vitali's covering Lemma, Differentiation of monotonic functions, Functions of bounded variation and its representation as difference of monotonic functions, Differentiation of Indefinite Integral, Fundamental Theorem of Calculus, Absolutely continuous functions and their properties. L spaces, convex functions, Jensen's inequalities, Measure space, Generalised Fatou, Lemma, Measure and outer measure, Extension of a measure, Caratheodory Extension Theorem.

TOPOLOGY

PAPER CODE: MM1003

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit-I

Definition and examples of topological spaces, closed sets and closure, dense subsets. Neighbourhoods interior, Exterior and boundary operations, Accumulation points and Derived sets. Bases and subbase. Subspaces and relative topology. Alternative method of defining a topology in terms of Kuratowski closure operator and neighbourhood systems. Continuous functions and homeomorphisms. Connected spaces. Connectedness on the real line. Components, Locally connected spaces.

Unit-II

Compactness, continuous functions and compact sets. Basic properties of compactness and finite intersection property. Sequentially and countably compact sets, Local compactness and one point compactification. Separation axioms T_0 , T_1 and T_2 spaces, Their characterisation and basic properties, Convergence on T_0 spaces first and second countable spaces, Lindelof's Theorems, Separable spaces and separability.

Unit-III

Regular and normal spaces, Urysohn's Lemma and Tietze Extension Theorem, T_3 and T_4 spaces, Complete regularity and complete normality, T/A and T_s spaces. Embedding and Metrization. Embedding Lemma and Tychonoff embedding, Urysohn's Metrization Theorem.

Unit-IV

Product topological spaces, Projection mappings, Tychonoff product topology in terms of standard subbases and its characterisation, Separation axioms and product spaces, Connectedness, locally connectedness and Compactness of product spaces. Product space as first axiom space. Nets and filters. Topology and convergence of nets. Hausdorffness and nets. Compactness and nets. Filters and their convergence. Canonical way of converting nets to filters and vice-versa, ultra filters and compactness. Stone-Cech compactification.

Unit-V

Homotopy of paths, Fundamental group, Covering spaces, The fundamental group of the circle and fundamental theorem of algebra. Covering of a space, local finiteness, paracompact spaces, Michael's theorem on characterisation of paracompactness in regular space, Paracompactness as normal, Nagata-Smirnov Metrization theorem.

PROGRAMMING IN C

PAPER CODE: MM1004

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit I

An overview of programming. Programming language, Classification. C Essentials-Program Development. Functions. Anatomy of a C function. Variables and Constant. Expressions. Assignment Statements. Formatting Source Files. Continuation Character. The Preprocessor. Scalar Data Types Declarations, Different Types of Integers. Different kinds of Integer Constants. Floating- Point Types. Initialisation. Mixing Types. Explicit Conversions—Casts. Enumeration Types. The Void Data Type, Typedefs. Finding the Address of an object. Pointers. Control Flow-Conditional Branching. The Switch Statement. Looping. Nested Loops. The break and continue Statements. The goto statement. Infinite Loops.

Unit II

Operators and Expressions — Precedence and Associativity. Unary Plus and Minus operators. Binary Arithmetic Operators. Arithmetic Assignment Operators. Increment and Decrement Operators. Comma Operator. Relational Operators. Logical Operators. Bit- Manipulation Operators. Bitwise Assignment Operators. Cast Operator. Size of Operators. Conditional Operators. Memory Operators.

Unit III

Arrays and Pointers — Declaring an Array. Arrays and Memory, Initialising Arrays, Encryption and Decryption. Pointer Arithmetic. Passing Pointers as Function Arguments. Accessing Array Elements through Pointers. Passing Arrays as Function Arguments. Sorting Algorithms. Strings. Multidimensional Arrays. Arrays of Pointers. Pointers to Pointers.

Unit IV

Strong Classes — Fixed vs. Automatic Duration. Scope. Global variables. The register Specifier. ANSI rules for the syntax and Semantics of the storage — class keywords. Dynamic Memory Allocation. Structures and Unions-Structures. Linked Lists. Unions. Enum Declarations. Functions — Passing Arguments. Declarations and Calls. Pointers to Functions. Recursion. The main () Function. Complex Declarations.

Unit V

The C Preprocessor—Macro Substitution. Conditional Compilation. Include Facility. Line Control. Input and Output—Streams, Buffering. The <Stdio. H> header File. Error Handling. Opening and Closing a File. Reading and Writing Data/Selecting an I/O Method. Unbuffered I/O Random Access. The standard library for input/output.

DIFFERENTIAL EQUATIONS

PAPER CODE: MM1005

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit I

Linear integral equations, some basic identities, initial value problems reduced to Volterra integral equations, Methods of successive substitutions and successive approximation to solve Volterra integral equations of second kind. Iterated kernels and Neumann series for Volterra equations. Resolvent kernel as a series in 1, Laplace transform method for a difference kernel, Solution of a Volterra integral equation of the first kind.

Boundary value problems reduced to Fredholm integral equations, methods of successive approximation and successive substitutions to solve Fredholm equations of second kind. Iterated kernels and Neumann series for Fredholm equations. Resolvent kernel as a sum of series. Fredholm resolvent kernel as a ratio of two series. Fredholm equations with separable kernels, approximation of a kernel by a separable kernel, Fredholm Alternative. Green's function, use of method of variation of parameters to construct the Green's function for a non homogeneous linear second order BVP, Basic four

properties of the Green's function, Alternate procedure for construction of the Green's function by using its basic four properties. Reduction of a BVP to a Fredholm integral equation with kernel as Green's function, Hilbert-Schmidt theory for symmetric kernels. (Relevant topics from Jerri's book).

Unit II

-approximate solution, Cauchy-Euler construction of an ϵ -approximate solution, Equicontinuous family of functions, Ascoli- Arzela lemma, Cauchy-Peano existence theorem. Uniqueness of solutions, Lipschitz condition, Picard-Lindelof existence and uniqueness theorem for $dt \ dy = f(t,y)$, solution of initial-value problems by Picard method. Sturm-Liouville BVPs, Sturm's separation and comparison theorems, Lagrange's identity and Green's formula for second order differential equations, properties of eigenvalues and eigenfunctions, Pruffer transformation, adjoint systems, self-adjoint equations of second order.

Linear systems, Matrix method for homogeneous first order system of linear differential equations, fundamental set and fundamental matrix, Wronskian of a system, Method of variation of constants for a non homogeneous system with constant coefficients, nth order differential equation equivalent to a first order system (Relevant topics from the books by Ross, and Coddington and Levinson).

Unit III

Nonlinear differential system, plane autonomous systems and critical points, classification of critical points – rotation points, foci, nodes, saddle points. Stability, asymptotical stability and unstability of critical points, almost linear systems, Liapunov function and Liapunov's method to determine stability for nonlinear systems.

Periodic solutions and Floquet theory for periodic systems, limit cycles, Bendixson non-existence theorem, Poincare-Bendixson theorem (Statement only), index of a critical point. (Relevant topics from the books by Ross, and Coddington and Levinson).

Unit IV

Motivating problems of calculus of variations, shortest distance, minimum surface of revolution, Branchistochrone problem, isoperimetric problem, geodesic. Fundamental lemma of calculus of variations, Euler's equation for one dependant function and its generalization to 'n' dependant functions and to higher order derivatives, conditional extremum under geometric constraints and under integral constraints (Relevant topics from the book by Gelfand and Fomin).

M.Sc. (Final)

INTEGRATION THEORY AND FUNCTIONAL ANALYSIS

PAPER CODE: MM2001

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit I

Signed measure, Hahn decomposition theorem, Jordan decomposition theorem, Mutually singular measure, Radon-Nikodym theorem. Lebesgue decomposition, Lebesgue-Stieltjes integral, Product measures, Fubini's theorem. Baire sets, Baire measure, Continuous functions with compact support, Regularity of measures on locally compact support, Riesz-Markoff theorem.

Unit II

Normed linear spaces, Metric on normed linear spaces, Holder's and Minkowski's inequality, Completeness of quotient spaces of normed linear spaces. Completeness of \mathbb{R} , \mathbb{C} , \mathbb{R}_n , \mathbb{C}_n and $C[a, b]$. Bounded linear transformation. Equivalent formulation of continuity. Spaces of bounded linear transformations, Continuous linear functional, Conjugate spaces, Hahn-Banach extension theorem (Real and Complex form), Riesz Representation theorem for bounded linear functional on V and $C[a, b]$.

Unit III

Second conjugate spaces, Reflexive spaces, Uniform boundedness principle and its consequences, Open mapping theorem and its application, projections, Closed Graph theorem, Equivalent norms, weak and strong convergence, their equivalence in finite dimensional spaces.

Unit IV

Compact operations and its relation with continuous operator. Compactness of linear transformation on a finite dimensional space, properties of compact operators, Compactness of the limit of the sequence of compact operators. The closed range theorem. Inner product spaces, Hilbert spaces, Schwarz's inequality, Hilbert space as normed linear space, Convex set in Hilbert spaces. Projection theorem.

Unit V

Orthonormal sets, Bessel's inequality, Parseval's identity, Conjugate of Hilbert space, Riesz representation theorem in Hilbert spaces. Adjoint of an operator on a Hilbert space, Reflexivity of Hilbert space, Self-adjoint operator, Positive operator, Normal and unitary operators, Projections on Hilbert space, Spectral theorem of finite dimensional spaces, Lax-Milgram theorem.

PARTIAL DIFFERENTIAL EQUATIONS AND MECHANICS

PAPER CODE: MM2002

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit I

Solution of three-dimensional Laplace equation by using the method of separation of variables in terms of Cartesian, cylindrical and spherical coordinates. Method of separation of variables to solve three-dimensional wave equation in Cartesian and spherical coordinates. Use of the method of separation of variables to find steady-state temperature in a rectangular plate, in a disk, in a bar with ends at different temperatures, in a semi-infinite bar, in an infinite plate, in an infinite cylinder, in a solid sphere (Relevant topics from the books by Sneddon, and O'Neil).

Unit II

Kinematics of a rigid body rotating about a fixed point, Euler's theorem, general rigid body motion as a screw motion, moving Coordinate system — rectilinear moving frame, rotating frame of reference, rotating earth.

Moments and products of inertia, Angular momentum of a rigid body, principal axes and principal moment of inertia of a rigid body, kinetic energy of a rigid body rotating about a fixed point, Momental ellipsoid and equimomental systems, coplanar mass distributions, general motion of a rigid body. Two- dimensional rigid body dynamics – problems illustrating the laws of motion and impulsive motion. (Relevant topics from the book of Chorlton).

Unit III

D'Alembert's principle: Constraints, holonomic and non-holonomic systems, Degree of freedom and Generalised coordinates, virtual displacement and virtual work, statement of principle of virtual work (PVW), possible velocity and possible acceleration, D' Alembert's principle.

Lagrangian Formulation : Ideal constraints, general equation of dynamics for ideal constraints, Lagrange's equations of the first kind, independent coordinates and generalised forces, Lagrange's equations of the second kind, generalized velocities and accelerations. Uniqueness of solution, variation of total energy for conservative fields. Lagrange's variable and Lagrangian function $L(t, q_i, \dot{q}_i)$, Lagrange's equations for potential forces, generalised moment p_i , Hamiltonian variable and Hamiltonian function $H(t, q_i, p_i)$, Donkin's theorem, ignorable coordinates.

Unit IV

Hamilton canonical equations, Routh variables and Routh function R, Routh's equations, Poisson Brackets and their simple properties, Poisson's identity, Jacobi – Poisson theorem. Hamilton action and Hamilton's principle, Poincare – Carton integral invariant, Whittaker's equations, Jacobi's equations, Lagrangian action and the principle of least action. Canonical transformation, necessary and sufficient condition for a canonical transformation, univalent Canonical transformation, free canonical transformation, Hamilton-Jacobi equation, Jacobi theorem, method of separation of variables in HJ equation, Lagrange brackets, necessary and sufficient conditions of canonical character of a transformation in terms of Lagrange brackets, Jacobian matrix of a canonical transformation, conditions of canonicity of a transformation in terms of Poisson brackets, Invariance of Poisson Brackets under canonical transformations.

Books Recommended

F. Gantmacher, Lectures in Analytic Mechanics, MIR Publishers, Moscow, 1975.

P.V. Panat, Classical Mechanics, Narosa Publishing House New Delhi, 2005.

N.C. Rana and P.S. Joag, Classical Mechanics, Tata McGraw- Hill. New Delhi, 1991.

Louis N. Hand and Janet D. Finch, Analytical Mechanics, CUP, 1998.

Sneddon, I.N., Elements of Partial Differential Equations. New York: McGraw Hill.

O'Neil, Peter V., Advanced Engineering Mathematics, ITP.

F. Chorlton, Textbook of Dynamics, CBS Publishers. New Delhi.

H.F. Weinberger, A First Course in Partial Differential Equations. John Wiley & Sons 1965.

K. Sankra Rao, Classical Mechanics. Prentice Hall of India, 2005.

M.R. Speigal, Theoretical Mechanics, Schaum Outline Series.

COMPLEX ANALYSIS

PAPER CODE: MM2003

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit I

Analysis functions, Cauchy-Riemann equation in cartesian and polar coordinates. Complex integration. Cauchy-Goursat Theorem. Cauchy's integral formula. Higher order derivatives. Morera's Theorem. Cauchy's inequality and Liouville's theorem, The fundamental theorem of algebra. Taylor's theorem.

Unit-II

Isolated singularities. Meromorphic functions. Maximum modulus principle. Schwarz lemma. Laurent's series. The argument principle. Rouche's theorem. Inverse function theorem. Residues. Cauchy's residue theorem. Evaluation of integrals. Branches of many valued functions with special reference to $\arg z$, $\log z$ and z^{λ} .

Unit-III

Bilinear transformations, their properties and classifications. Definitions and examples of Conformal mappings. Space of analytic functions. Hurwitz's theorem. Montel's theorem. Riemann mapping theorem. Weierstrass' factorisation theorem. Gamma function and its properties. Riemann Zeta function. Riemann's functional equation. Runge's theorem. Mittag-Leffler's theorem.

Unit IV

Analytic Continuation. Uniqueness of direct analytic continuation. Uniqueness of analytic continuation along a curve. Power series method of analytic continuation. Schwarz Reflection principle. Monodromy theorem and its consequences. Harmonic functions on a disk. Harnack's inequality and theorem. Dirichlet problem. Green's function. Canonical products. Jensen's formula. Poisson-Jensen formula. Hadamard's three circles theorem.

Unit V

Order of an entire function. Exponent of Convergence. Borel's theorem. Hadamard's factorisation theorem. The range of an analytic function. Bloch's theorem. The Little Picard theorem. Schottky's theorem. Montel Caratheodory and the Great Picard theorem. Univalent functions. Bieberbach's conjecture (Statement only) and the $1/4$ theorem.

Group-I (Pure Group) **ADVANCED DISCRETE MATHEMATICS** **PAPER CODE: MM2004**

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit I

Formal Logic: Statement, Symbolic representation, tautologies, quantifiers, predicates and validity, propositional logic. Semigroups and Monoids: Definitions and examples of semigroups and monoids (including those pertaining to concentration operations). Homomorphism of semigroups and monoids, Congruence relation and quotient semigroups, sub semigroups and sub monoids, Direct products basic homomorphism theorem. Lattices: Lattices as partially ordered sets, their properties. Lattices and algebraic systems. Sub lattices, direct products and homomorphism. Some special lattices for example complemented and distributive lattices.

Unit II

Boolean Algebra: Boolean Algebra as Lattices. Various Boolean Identities, Join-irreducible elements. Atoms and Minterms. Boolean Forms and their Equivalence. Minterm Boolean Forms, Sum of Products Canonical Forms. Minimisation of Boolean Functions. Applications of Boolean Algebra to Switching Theory (using AND, OR and NOT gates). The Karnaugh Map method.

Unit III

Graph Theory — Definition of (undirected) Graphs, Paths, Circuits, Cycles and Subgraphs. Induced Subgraphs. Degree of a vertex. Connectivity. Planar Graphs and their properties. Trees, Duler's Formula for connected Planar Graphs, Complete and Complete Bipartite Graphs. Kuratowski's Theorem (statement only) and its use. Spanning Trees. Cut-sets. Fundamental Cut-sets and Cycles/Minimal Spanning Trees and Kruskal's Algorithm. Matrix Representations of Graphs. Euler's Theorem on the Existence of Eulerian Paths and Circuits, Directed Graphs. Indegree and Outdegree of a Vertex. Weighted undirected Graphs. Dijkstra's Algorithm. Strong Connectivity and Warshall's Algorithm. Directed Trees. Search Trees. Tree Traversals.

Unit IV

Introductory Computability Theory — Finite state machines and their transition table diagrams. Equivalence of finite state machines. Reduced Machines, Homomorphism. Finite automata. Acceptors. Non-deterministic finite automata and Equivalence of its power to that of Deterministic Finite Automata. Moore and Mealy Machines.

Unit V

Grammar and Languages — Phrase Structure Grammars. Rewriting Rules. Derivations Sentential Forms. Language generated by Grammar. Regular, Context Free, and Context Sensitive Grammar and Languages. Regular sets, Regular Expressions and the Pumping Lemma, Kleene's Theorem. Notions of Syntax Analysis. Polish Notations. Conversion of Infix Expressions to Polish Notations. The Reverse Polish Notation.

ANALYTICAL NUMBER THEORY

PAPER CODE: MM2005

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit I

Primes in certain arithmetical progressions, Fermat numbers and Mersenne numbers. Farey series and some results concerning Farey series. Approximation of irrational numbers by rationals. Hurwitz's theorem on irrationality of e and n . The series of Fibonacci and Lucas. System of linear congruences, Chinese Remainder Theorem. Congruence to prime power modulus.

Unit II

Quadratic residues and non-residues. Legendre's Symbol. Gauss Lemma and its applications. Quadratic Law of Reciprocity. Jacobi's Symbol. The arithmetic in \mathbb{Z} . The group U_n . Primitive roots. The group U_p (p -odd) and U_{2n} . The existence of primitive roots. The group of quadratic residues. Quadratic residues for prime power moduli and arbitrary moduli.

Unit III

Riemann Zeta Function $\zeta(s)$ and its convergence. Application to prime numbers. $\zeta(s)$ as Euler's product. Evaluation of $\zeta(2)$ and $\zeta(2k)$. Dirichlet series with simple properties. Dirichlet series as analytic function and its derivative. Euler's products. Introduction to modular forms.

Unit IV

Diophantine equations, $x^2 + y^2 = z^2$ and $x^4 + y^4 = z^4$. The representation of number by two or four squares. Waring's problem. Four square theorem. The number $g(k)$ and $G(k)$. Lower bounds for $g(k)$ and $G(k)$.

Algebraic number and Integers; Gaussian integers and its properties. Primes and fundamental theorem in the ring of Gaussian integers. Integers and fundamental theorem in $\mathbb{Q}(\omega)$ where $\omega^3 = 1$, algebraic fields. Primitive polynomials. The general quadratic field $\mathbb{Q}(\sqrt{m})$, Units of $\mathbb{Q}(\sqrt{2})$. Fields in which fundamental theorem is false. Real and complex Euclidean fields. Fermat's theorem in the ring of Gaussian integers. Primes of $\mathbb{Q}(2)$ and $\mathbb{Q}(5)$. Luca test for the normality of the Mersenne number.

Unit V

Arithmetical function $\phi(n)$, $u(n)$, $d(n)$ and $a(n)$, Mobius inversion formulae. Perfect numbers. Order and average order of $d(n)$, $\phi(n)$. The functions $\sigma(x)$, $\sigma_2(x)$ and $A(x)$. Bertrand postulate. Sum $\sum_{p \leq x} \frac{1}{p}$ and product $\prod_{p \leq x} (1 + \frac{1}{p})$. Mertens's theorem. Selberg's theorem. Prime number Theorem.

Group-II (Applied Group)

MECHANICS OF SOLIDS

PAPER CODE: MM2006

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

Unit I

Analysis of Stress: Affine transformation. Infinitesimal affine deformation. Geometrical interpretation of the components of strain. Strain quadric of Cauchy. Principal strains and invariants. General infinitesimal deformation. Saint-Venant's equations of Compatibility. Finite deformations.

Unit II

Equations of Elasticity: Generalised Hooke's law. Homogeneous isotropic media. Elasticity moduli for isotropic media. Equilibrium and dynamic equations for an isotropic elastic solid. Strain energy function and its connection with Cauchy's law. Uniqueness of solution. Beltrami-Micheli compatibility equations. Saint-Venant's principle.

Unit III

Two - dimensional Problems: Plane stress. Generalised plane stress. Airy stress function. General solution of Biharmonic equation, Stresses and displacements in terms of complex potentials. Simple problems. Stress function appropriate to problems of plane stress. Problems of semi-infinite solids with displacements or stresses in prescribed or plane boundary.

Unit IV

Torsional Problem: Torsion of cylindrical bars. Torsional rigidity. Torsion and stress functions. Lines of shearing stress. Sinfteproblems related to circle, ellipse and equilateral triangle. Variation in solids: Theorems of minimum potential energy. Theorems of minimum complementary energy. Reciprocal theorem of Betti and Rayleigh. Deflection of elastic string, central line of a beam and elastic membrane. Torsional cylinders.

Variational problem related to Biharmonic equation. Solution of Euler's equation by Ritz, Galerkinaed Kantorovich methods.

Unit V

Elastic Waves: Propagation of waves in an isotropic elastic solid medium. Waves of dilatation and distortion Plane waves. Elastic surface waves such as Rayleigh and Love waves.

FLUID DYNAMICS **PAPER CODE: MM2007**

Marks: 100

Time: 3Hrs

Note: Examiner will be required to set NINE questions in all. Question No. 1 will be compulsory which consists of 12 short-answer type questions each of 2 marks covering the entire syllabus out of which candidate will be required to attempt ten questions. In addition to Q.No. 1, candidate will be required to attempt four more questions from the remaining eight questions each carrying 20 marks.

UNIT I

Kinematics — Lagrangian and Eulerian methods. Equation of continuity. Boundary surface. Stream lines. Path lines and streak lines. Velocity potential. Irrotational and rotational motions. Vortex lines. Equations of Motion—Lagrange's and Euler's equations of motion. Bernoulli's theorem. Equation of motion by flux method. Equations referred to moving axes, Impulsive actions. Stream function. Irrotational motion in two-dimensions. Complex velocity potential. Sources, sinks, doublets and their images. Conformal mapping, Milne-Thomson circle theorem. Two-dimensional irrotational motion produced by motion of circular, co-axial and elliptic cylinders in an infinite mass of liquid. Kinetic energy of liquid. Theorem of Blasius. Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere. Equation of motion of a sphere. Stoke's stream function.

UNIT II

Vortex motion and its elementary properties. Kelvin's proof of permance. Motions due to circular and rectilinear vortices. Wave motion in a gas. Speed of Sound. Equation of motion of a gas. Subsonic, sonic and supersonic flows of a gas. Isentropic gas flows. Flow through a nozzle. Normal and oblique shocks.

UNIT III

Stress components in a real fluid. Relations between rectangular components of stress. Connection between stresses and gradients of velocity. Navier-stoke's equations of motion. Plane Poiseuille and Couette flows between two parallel plates. Theory of Lubrication. Flow through tubes of uniform cross section in form of circle, annulus, ellipse and equilateral triangle under constant pressure gradient. Unsteady flow over a flat plate.

UNIT IV

Dynamical similarity. Buckingham p-theorem. Reynolds number. Prandtl's boundary layer. Boundary layer equations in twodimensions. Blasius solution. Boundary-layer thickness. Displacement thickness. Karman integral conditions. Separations of boundary layer flow.